Are you ready for Beast Academy 4B?

**Step 1.** The student should try to answer every question without a calculator and without help.

**Step 2.** Check the student’s answers using the solutions at the end of this document.

**Step 3.** The student should be given a second chance on problems that he or she answered incorrectly.

1. Label point S on the dot grid below so that quadrilateral BEST is a parallelogram.

   B  •  •  •  •  •  •  •  •  E
     1
   .  •  •  •  •  •  •  •  .
   .  •  •  •  •  •  •  •  .
   T  •  •  •  •  •  •  •  •

2. Label point R on the dot grid below so that quadrilateral GRID is a kite.

   G  •  •  •  •  •  •  •  •  •
   .  •  •  •  •  •  •  •  .
   .  •  •  •  •  •  •  •  .
   D  •  •  •  •  •  •  •  •
   I  •  •  •  •  •  •  •  •

3. The measure of angle AXB is twice the measure of angle BXC. Without using a protractor, find the measure of angle BXC.

4. Shade the square that could be removed from the shape below to leave a shape with at least one **line** of symmetry.

5. Draw a cut line along the grid to split the shape into two pieces so that both pieces have **rotational** symmetry.
Compute each product.

6. \( 9 \times 327 = \) ______
7. \( 27 \times 64 = \) ______
8. \( 17 \times 2,003 = \) ______
9. \( 2,020 \times 701 = \) ______

Rewrite each multiplication expression as a power.

10. \( 2 \times 2 \times 2 \times 2 = \) ______
11. \( 13 \times 13 \times 13 \times 13 \times 13 = \) ______

Compute the value of each power.

12. \( 8^2 = \) ______
13. \( 5^4 = \) ______

Fill in the missing exponent in each equation.

14. \( 2^3 \times 2^7 = 2^{\square} \)
15. \( 7 \times 7^{\square} = 7^5 \)

Answer each question below.

16. How can the number 80 be written as the sum of two powers of 2?
16. \( 80 = \) _____ + _____.

17. The product 147 \times 594 \times 912 ends with the digit ______.

18. Write \( 49^3 \) as a power of 7.
18. _________
1. A parallelogram is a four-sided polygon with two pairs of opposite sides that are parallel. We draw parallelogram BEST by connecting B to E to S to T to B. So, segments BE and TB are sides of the parallelogram.

Then, we draw a line parallel to segment BE that passes through point T, and a line parallel to segment BT that passes through point E. We label the intersection of these two points “S” to make parallelogram BEST.

2. We draw kite GRID by connecting G to R to I to D to G. So, segments ID and DG are sides of the kite.

Kites have two pairs of adjacent sides that are congruent, but segments ID and DG are not the same length. So, segment GR is the same length as segment DG, and segment RI is the same length as segment ID.

There are two other segments in the grid with endpoint G that has the same length as segment DG:

Only one of these endpoints will make a kite when its other endpoint is connected to point I, so we label that point R.

3. The sum of the measures of angles AXB and BXC equals the measure of straight angle AXC. So, the sum of the measures of angles AXB and BXC is 180°.

We are given that the measure of angle AXB is twice the measure of angle BXC. So, if we let $a$ represent the measure in degrees of angle BXC, then the measure of angle AXB is $(a+a)$°.

Since the sum of the measures of angles AXB and BXC is 180 degrees, we have $a + (a+a) = 180$.

We simplify the left side to rewrite the equation as $3a = 180$. Since $3 \times 60 = 180$, we have $a = 60$.

So, the measure of angle BXC is 60°.

4. If we remove the shaded square, then we are left with the figure that has the line of symmetry shown below:

5. If we make the cut shown, then we create two shapes, each with rotational symmetry.

Both shapes have rotational symmetry of order 2.

6. We set up the numbers as shown.

Distributing the 9 gives three partial products: $9 \times 7 = 63$, $9 \times 20 = 180$, and $9 \times 300 = 2,700$.

We stack the partial products so that the units, tens, and hundreds digits line up, as shown below.

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Then, we add the partial products:

\[
\begin{array}{c}
327 \\
\times 9 \\
\hline
63 \\
180 \\
+ 2,700 \\
\hline
2,943 \\
\end{array}
\]

7. Each number represented by a digit in 27 is multiplied by each number represented by a digit in 64.

Distributing gives four partial products, \(4 \times 7 = 28\), \(4 \times 20 = 80\), \(60 \times 7 = 420\), and \(60 \times 20 = 1,200\).

\[
\begin{array}{cccc}
27 & 27 & 27 & 27 \\
\times 64 & \times 64 & \times 64 & \times 64 \\
\hline
28 & 28 & 28 & 28 \\
80 & 80 & 80 & 80 \\
420 & 420 & 420 & 420 \\
\hline
& & & 1,200 \\
\end{array}
\]

Then, we add the partial products.

\[
\begin{array}{c}
27 \\
\times 64 \\
\hline
28 \\
80 \\
420 \\
+ 1,200 \\
\hline
1,728 \\
\end{array}
\]

8. First, we distribute the 7 in 17. This gives us four partial products: \(7 \times 3 = 21\), \(7 \times 0 = 0\), \(7 \times 0 = 0\) and \(7 \times 2,000 = 14,000\).

It is not necessary to write a partial product of 0, since adding 0 does not change a sum.

\[
\begin{array}{cccc}
2,003 & 2,003 & 2,003 & 2,003 \\
\times 17 & \times 17 & \times 17 & \times 17 \\
\hline
21 & 21 & 21 & 21 \\
\hline
& & & 14,000 \\
\end{array}
\]

Next, we distribute the 1 that is the tens digit of 17. Since we know that \(10 \times 2,003 = 20,030\) we do not need to distribute the 10 to each digit in 2,003 separately.

\[
\begin{array}{c}
2,003 \\
\times 17 \\
\hline
21 \\
14,000 \\
+ 20,030 \\
\hline
34,051 \\
\end{array}
\]

9. We distribute as shown below. We do not need to write a partial product of 0.

\[
\begin{array}{ccc}
2,020 & 2,020 & 2,020 \\
\times 701 & \times 701 & \times 701 \\
\hline
2,020 & 2,020 & 2,020 \\
\hline
\end{array}
\]

\[
\begin{array}{ccc}
2,020 & 2,020 & 2,020 \\
\times 701 & \times 701 & \times 701 \\
\hline
2,020 & 2,020 & 2,020 \\
14,000 & 14,000 & 14,000 \\
\hline
1,400,000 \\
\end{array}
\]

Finally, we add the partial products.

\[
\begin{array}{c}
2,020 \\
\times 17 \\
\hline
2,020 \\
14,000 \\
+ 1,400,000 \\
\hline
1,416,020 \\
\end{array}
\]

10. We multiply four 2's, so 2 is the base and 4 is the exponent: \(2 \times 2 \times 2 \times 2 = 2^4\).

11. We multiply five 13's, so 13 is the base and 5 is the exponent: \(13 \times 13 \times 13 \times 13 \times 13 = 13^5\).

12. We evaluate \(8^2\) by multiplying two 8's:

\[
8^2 = 8 \times 8 = 64.
\]

13. We evaluate \(5^4\) by multiplying four 5's:

\[
5^4 = 5 \times 5 \times 5 \times 5 = 625.
\]

14. We write the powers as products:

\[
2^3 \times 2^7 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2.
\]

All together, we multiply ten 2's, which equals 2 to the 10th power: \(2^1 \times 2^2 = 2^{1+2} = 2^3\).

--- or ---

When we evaluate \(2^1 \times 2^2\), we multiply three 2's and then another seven 2's. All together, we multiply 3 + 7 = 10 twos, which is equal to \(2^{10}\). So, \(2^3 \times 2^7 = 2^{10}\).

15. The product 7 \(\times 7^0\) is equal to 7. 

7 is the product of five 7's. So, we need to multiply 7 by four more 7's to get 7^5. Therefore, \(7 \times 7^7 = 7^5\).

16. We look at the powers of 2 between 1 and 80:

\[
2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 8, \quad 2^4 = 16, \quad 2^5 = 32, \quad \text{and} \quad 2^6 = 64.
\]

16 + 64 = 80, and the sum of any other pair is smaller or larger than 80. So, the only two powers of 2 that we can add to get 80 are \(2^4 = 16\) and \(2^6 = 64\).

So, we write \(80 = 2^4 + 2^6\).
Since addition is commutative, you may have instead written \(80 = 2^4 + 2^5\).

17. The units digit of \(147 \times 594 \times 912\) is the same as the units digit of \(7 \times 4 \times 2 = 56\).
   So, the product \(147 \times 594 \times 912\) ends with the digit 6.
   In fact, \(147 \times 594 \times 912 = 79,634,016\).

18. We know \(49^3 = 49 \times 49 \times 49\). Since \(49 = 7 \times 7\), we replace each 49 with \(7 \times 7\) to get
   \[
   49^3 = 49 \times 49 \times 49 = (7 \times 7) \times (7 \times 7) \times (7 \times 7).
   \]
   So, \(49^3\) is the product of six 7's. Therefore, \(49^3 = 7^6\).