Practice 5D: Chapter 10, page 21, problems #105-106

In a **Percent Square** puzzle, the goal is to fill every empty square in the grid according to the following rules:

- Each square must contain a single positive digit.
- The percent next to a row or above a column gives the percent of the row's or column's sum that is in its shaded square(s).

105. ★★

		25%	60%	70%
90%				
37.5%				
70%				
30%				

106. ★★

	40%	20%	30%	20%
25%				5
			5	
18.75%		1		
$33\frac{1}{3}$%	2			

Percent Squares Sol'n, #105-106

Practice 5D: Chapter 10, page 21, problems #105-106

In the 4-by-4 puzzles, we only consider denominators (row or column sums) from 1+1+1+1 = 4 to 9+9+9+9 = 36.
If a row or column has exactly 2 shaded or unshaded squares, we know the sum in those squares is from 1+1 = 2 to 9+9 = 18.

If a row or column has exactly 3 shaded or unshaded squares, we know the sum in those squares is from 1+1+1 = 3 to 9+9+9 = 27.

105. In the top row, $90\% = \frac{9}{10} = \frac{18}{20}$ of the sum is in the shaded squares. Therefore, the sum of the top row is 10 or 20.

- If the sum of the top row is 10, then the sum of the shaded squares is 9 and the sum of the unshaded squares is 1. However, we cannot add two positive digits to get a sum of 1. ✗
- If the sum of the top row is 20, then the sum of the shaded squares is 18, and the sum of the unshaded squares is $20 - 18 = 2$. ✓

There is only one way to get a sum of 18 from two digits: 9+9. There is also only one way to get a sum of 2 from two positive digits: 1+1.

So, we complete the top row as shown.

		25%	60%	70%
90%	1	1	9	9
37.5%				
70%				
30%				

In the right column, $70\% = \frac{7}{10} = \frac{14}{20}$ of the sum is in the shaded squares. One shaded square in this column is 9. We cannot make a sum of 7 by adding a digit to 9.

So, the sum of the right column is 20 and the sum of the shaded squares is 14. The empty shaded square is $14 - 9 = 5$.

		25%	60%	70%
90%	1	1	9	9
37.5%				
70%				5
30%				

In the row highlighted below, $70\% = \frac{7}{10} = \frac{14}{20} = \frac{21}{30}$ of the sum is in the shaded squares.

The remaining $30\% = \frac{3}{10} = \frac{6}{20} = \frac{9}{30}$ of the sum is in the unshaded square. So, the unshaded square in this row is 3, 6, or 9.

		25%	60%	70%	
90%	1	1	9	9	
37.5%					
70%				5	3, 6, or 9
30%					

Since the 9 in the top square of the column highlighted below is $60\% = \frac{9}{15}$ of the sum of this column, the sum of this column is 15. The sum of the three unshaded squares in this column is therefore $15 - 9 = 6$.

There are just two ways to get a sum of 6 from three positive digits: 2+2+2 and 1+2+3.

In a previous step, we found that the square above the bottom row in the highlighted column can only be 3, 6, or 9.

So, the three unshaded squares in this column are 1, 2, and 3. We place the 3 as shown and note that the other two squares in the column are 1 and 2.

		25%	60%	70%	
90%	1	1	9	9	
37.5%					
70%			3	5	1+2
30%					

Since the 3 in the row highlighted below row is $30\% = \frac{3}{10}$ of this row's sum, the sum of this row is 10.

So, the sum of the two empty squares in this row is $10 - 3 - 5 = 2$. The only way to get a sum of 2 from two positive digits is 1+1. So, we complete this row as shown.

		25%	60%	70%	
90%	1	1	9	9	
37.5%					
70%	1	1	3	5	1+2
30%					

Percent Squares #105-106, Solution (cont'd)

In the right column, $70\% = \frac{14}{20}$ of the sum is in the shaded squares. So, the sum of the unshaded squares is $20-9-5=6$.

There are three ways to make a sum of 6 from two positive digits: $1+5$, $2+4$, or $3+3$.

	25%	60%	70%
90%	1	1	9	9
37.5%				
70%	1	1	3	5
30%				

1+2

1+5, 2+4, or 3+3.

In the bottom row, $30\% = \frac{3}{10} = \frac{6}{20} = \frac{9}{30}$ of the sum is in the shaded squares. The remaining $70\% = \frac{7}{10} = \frac{14}{20} = \frac{21}{30}$ of the sum is in the unshaded squares.

From the digits that can be placed in the unshaded squares, we see that the sum of the unshaded squares cannot be more than $2+5=7$.

Therefore, the sum of the bottom row is 10, and the sum of the unshaded squares is 7.

There is only one way to get a sum of 7 from the digits that can be placed in the unshaded squares: $2+5$. We place the 2 and 5 as shown.

	25%	60%	70%
90%	1	1	9	9
37.5%				
70%	1	1	3	5
30%			2	5

1+2

1+5, ~~2+4~~, or ~~3+3~~.

Then, we complete the two rightmost columns as shown.

	25%	60%	70%
90%	1	1	9	9
37.5%			1	1
70%	1	1	3	5
30%			2	5

1+~~2~~

1+~~5~~

In the row highlighted below, $37.5\% = \frac{3}{8} = \frac{6}{16} = \frac{9}{24}$ of the sum is in the shaded squares. Two 1's are already placed in this row, so the sum of this row cannot be more than $9+9+1+1=20$. We only consider $37.5\% = \frac{3}{8} = \frac{6}{16}$.

The remaining $62.5\% = \frac{5}{8} = \frac{10}{16}$ of the sum is in the unshaded squares.

So, the sum of the unshaded squares is 5 or 10, and the empty unshaded square in this row is $5-1-1=3$ or $10-1-1=8$.

	25%	60%	70%
90%	1	1	9	9
37.5%		8	1	1
70%	1	1	3	5
30%			2	5

3 or 8

In the column highlighted below, 25% of the sum of the column is in the shaded square.

The remaining $75\% = \frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$ of the sum is in the unshaded squares. So, the sum of these unshaded squares is 3, 6, 9, or 12.

There is already a 1 in one unshaded square. In a previous step, we saw that the empty unshaded square in this column can only be 3 or 8. So, the sum of these squares is $3+1=4$ or $8+1=9$.

Therefore, the sum of the unshaded squares is 9, and we place 8 in the empty unshaded square as shown.

	25%	60%	70%
90%	1	1	9	9
37.5%		8	1	1
70%	1	1	3	5
30%			2	5

We use the strategies discussed in previous problems to complete the puzzle as shown.

	25%	60%	70%
90%	1	1	9	9
37.5%	6	8	1	1
70%	1	1	3	5
30%			2	5

	25%	60%	70%
90%	1	1	9	9
37.5%	6	8	1	1
70%	1	1	3	5
30%		2	2	5

Final:

		25%	60%	70%
90%	1	1	9	9
37.5%	6	8	1	1
70%	1	1	3	5
30%	1	2	2	5

106. In the highlighted row, $18.75\% = \frac{3}{16} = \frac{6}{32}$ of the sum is in the shaded square.

The sum of this row cannot be more than $9+1+9+9=28$. So, the sum of this row is 16, and the shaded square is 3.

	40%	20%	30%	20%
25%				5
			5	
18.75%		1	3	
$33\frac{1}{3}\%$	2			

Since the 3 in the shaded square of the column highlighted below is $30\% = \frac{3}{10}$ of the sum of this column, the sum of this column is 10.

So, the sum of the two empty unshaded squares is $10-5-3=2$. The only way to make a sum of 2 from two positive digits is $1+1$. So, we complete this column as shown.

	40%	20%	30%	20%
25%			1	5
			5	
18.75%		1	3	
$33\frac{1}{3}\%$	2		1	

In the left column, $40\% = \frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$ of the sum is in the shaded square. So, the top-left square is 2, 4, 6 or 8.

	40%	20%	30%	20%
25%			1	5
			5	
18.75%		1	3	
$33\frac{1}{3}\%$	2		1	

2, 4, 6, or 8

In the top row, 25% of the sum is in the shaded square. Since this square is 2, 4, 6, or 8, we only consider $25\% = \frac{2}{8} = \frac{4}{16} = \frac{6}{24} = \frac{8}{32}$.

The remaining $75\% = \frac{6}{8} = \frac{12}{16} = \frac{18}{24} = \frac{24}{32}$ of the sum is in the unshaded squares. From the 1 and 5 already in this row, the sum of the unshaded squares is at least $1+1+5=7$, but not more than $9+1+5=15$.

So, the sum of this row is 16, and the sum of the unshaded squares is 12. Therefore, the shaded square is 4, and the empty unshaded square is $16-4-1-5=6$.

	40%	20%	30%	20%
25%	4	6	1	5
			5	
18.75%		1	3	
$33\frac{1}{3}\%$	2		1	

The 4 in the top-left square is $40\% = \frac{4}{10}$ of the sum of the left column. So, the sum of the left column is 10, and the sum of the two empty unshaded squares in this column is $10-4-2=4$. There are two ways to get a sum of 4 from two positive digits: $1+3$ and $2+2$.

	40%	20%	30%	20%
25%	4	6	1	5
			5	
18.75%		1	3	
$33\frac{1}{3}\%$	2		1	

1+3 or 2+2

In a previous step, we found that the sum of the row highlighted below is 16. So, the sum of the two empty unshaded squares in this row is $16-1-3=12$.

One of these empty unshaded squares is 1, 2, or 3. The only way to add a digit to 1, 2 or 3 and get a sum of 12 is $9+3$. So we complete this row as shown.

	40%	20%	30%	20%
25%	4	6	1	5
			5	
18.75%	3	1	3	9
$33\frac{1}{3}\%$	2		1	

1+3 or ~~2+2~~

Therefore, the empty square in the left column is 1.

	40%	20%	30%	20%
25%	4	6	1	5
	1		5	
18.75%	3	1	3	9
$33\frac{1}{3}\%$	2		1	

$1+\cancel{3}$

In the right column, 20% of the sum is in the shaded square. From the 5 and 9 already in this column, the sum of this column is at least $9+5+1+1=16$ but not more than $5+9+9+9=32$.

So, we only consider $20\% = \frac{4}{20} = \frac{5}{25} = \frac{6}{30}$. The shaded square is 4, 5, or 6.

	40%	20%	30%	20%
25%	**4**	**6**	**1**	5
	1		5	
18.75%	**3**	1	**3**	**9**
$33\frac{1}{3}\%$	2		**1**	

4, 5, or 6

In the column highlighted below, 20% of the sum is in the shaded square.

The remaining 80% of the sum is in the unshaded squares. From the 1 and 6 already in this row, the sum of the unshaded squares is at least $6+1+1=8$, but not more than $6+1+9=16$. Also, the total sum of this row cannot be more than $6+1+9+9=25$.

So, we only consider $80\% = \frac{8}{10} = \frac{12}{15}$.

Therefore, the sum of the unshaded squares in this column is 8 or 12, and the empty unshaded square is $8-6-1=1$ or $12-6-1=5$.

	40%	20%	30%	20%
25%	**4**	**6**	**1**	5
	1		5	
18.75%	**3**	1	**3**	**9**
$33\frac{1}{3}\%$	2		**1**	

1 or 5 4, 5, or 6

In the bottom row, only placing the 5 and 4 as shown puts $33\frac{1}{3}\% = \frac{4}{12}$ of the sum of the bottom row in the shaded square.

	40%	20%	30%	20%
25%	**4**	**6**	**1**	5
	1		5	
18.75%	**3**	1	**3**	**9**
$33\frac{1}{3}\%$	2	**5**	**1**	**4**

We use strategies discussed in previous problems to complete the puzzle as shown.

	40%	20%	30%	20%
25%	**4**	**6**	**1**	5
	1	**3**	5	**2**
18.75%	**3**	1	**3**	**9**
$33\frac{1}{3}\%$	2	**5**	**1**	**4**