EXAMPLE | What power of 7 is equal to $7^5 \div 7^3$?

We can use multiplication to solve division problems.

To divide 7^5 by 7^3, we look for the power of 7 we multiply by 7^3 to get 7^5.

Since $7^2 \cdot 7^3 = 7^5$, we know $7^5 \div 7^3 = \mathbf{7^2}$.

— ***or*** —

We can write the quotient as a fraction. $7^5 \div 7^3 = \frac{7^5}{7^3}$.

Writing the powers as products, we cancel three 7's in the numerator and denominator to get

$$\frac{7^5}{7^3} = \frac{\cancel{7} \cdot \cancel{7} \cdot \cancel{7} \cdot 7 \cdot 7}{\cancel{7} \cdot \cancel{7} \cdot \cancel{7}} = 7 \cdot 7 = \mathbf{7^2}.$$

As long as a is not zero, to divide a^m by a^n, we subtract the exponents and keep the same base.

$$a^m \div a^n = a^{m-n}. \text{ As a fraction, } \frac{a^m}{a^n} = a^{m-n}.$$

PRACTICE | Answer each question below.

34. What power of 5 is equal to $5^{18} \div 5^6$? **34.** __________

35. What power of 2 can we divide 2^6 by to get 2^2? **35.** __________

36. Circle the expression below that is ***not*** equal to 6^5.

$6^7 \div 6^2$ $\quad$ $6^2 \cdot 6^3$ $\quad$ $6^2 + 6^3$ $\quad$ $2^5 \cdot 3^5$ $\quad$ $\frac{6^{11}}{6^6}$

37. What power of 3 is equal to $\frac{3^{12}}{3^4 \cdot 3^3}$? **37.** __________

In a **Product Square** puzzle, the product of the three powers in each row and column match the product given above the square.

Below is a Product Square example and its solution.

5^9

5^2	5^3	
	5^4	
		5^3

➔

5^9

5^2	5^3	**5^4**
5^3	5^4	**5^2**
5^4	**5^2**	5^3

PRACTICE | Fill each missing entry in the Product Squares below with a power so that each row and column have the given product.

38. Write each entry as a power of 2.

2^{21}

2^6	2^6	
	2^8	2^8

39. Write each entry as a ***different*** positive power of 7.

7^{15}

7^6	7^7	
		7^4

40. ★ Write each entry as a perfect square.

120^2

	2^2	3^2
1^2		
		4^2

41. ★ Write each entry as a power of 2, 3, or 6 with a positive exponent.

$2^8 \cdot 3^6$

		3^4
	2^2	
		6^2

EXPONENTS
Zero and Negative

We can use **zero** as an exponent. Any power with an exponent of 0 is equal to 1. This works nicely with our exponent rules. For example,

$$7^4 \div 7^4 = 7^{4-4} = 7^0 = 1.$$

Exponents can also be **negative.**

We can use exponent rules to see how to define negative exponents. For example,

$$7^3 \div 7^5 = \frac{7^3}{7^5} = \frac{\cancel{7} \cdot \cancel{7} \cdot \cancel{7}}{7 \cdot 7 \cdot \cancel{7} \cdot \cancel{7} \cdot \cancel{7}} = \frac{1}{7 \cdot 7} = \frac{1}{7^2}.$$

Our exponent rules for division suggest that

$$7^3 \div 7^5 = 7^{3-5} = 7^{-2}.$$

So, we define $7^{-2} = \frac{1}{7^2}$.

As long as a is not zero, a^{-n} is used to represent the reciprocal of a^n.

$$a^{-n} = \frac{1}{a^n}.$$

PRACTICE | Answer each question below.

42. Write 5^{-2} as a fraction in simplest form. **42.** __________

43. Write 2^{-5} as a fraction in simplest form. **43.** __________

44. Write $\frac{1}{11^3}$ as a power of 11. **44.** __________

45. Write $\frac{1}{25^2}$ as a power of 5. **45.** __________

PRACTICE | Answer each question below.

46. Simplify $(5^4 \cdot 5^{-4}) + (4^3 \cdot 4^{-3}) + (3^2 \cdot 3^{-2})$.

46. __________

47. Order 4^{-3}, 3^{-4}, and 2^{-5} from least to greatest.

47. _______ < _______ < _______

48. Express $2^{-3} + 3^{-2}$ as a fraction in simplest form.

48. __________

49. Write $(2^{-2}) - (2^{-3}) - (2^{-4})$ as a fraction in simplest form.

49. __________

50. For how many integer values of n is 2^n between 3^{-1} and 3^{-3}?

50. __________

51. What power of 4 is equal to 8^{-2}?

51. __________

52. ★ Write $2^{-4} \cdot 6^2 \cdot 5^{-3}$ as a decimal.

52. __________