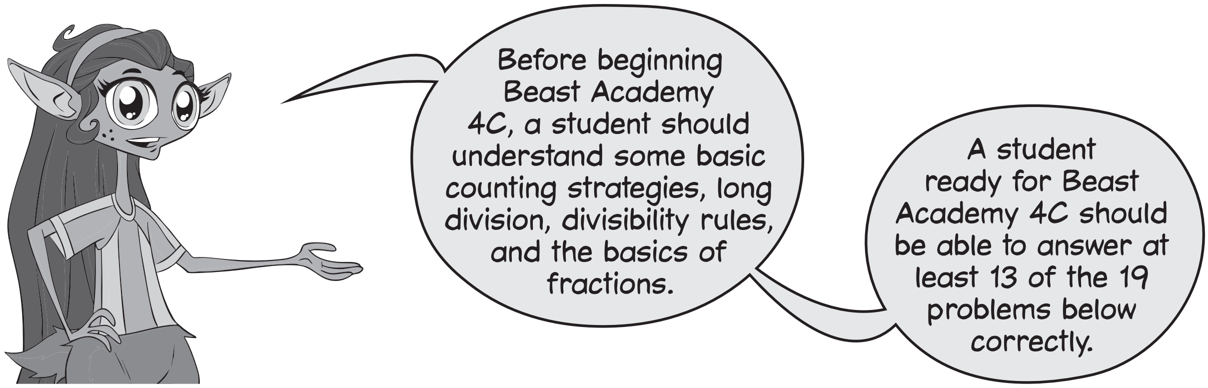
Are you ready for Beast Academy 4C?



Step 1. The student should try to answer every question without a calculator and without help.

Step 2. Check the student's answers using the solutions at the end of this document.

Step 3. The student should be given a second chance on problems that he or she answered incorrectly.

1. How many even numbers are there from 6 to 54? **1.** ________

6, 8, 10, ... , 50, 52, 54

2. How many three-digit numbers have three odd digits? **2.** ________

3. How many different arrangements of the letters in the word MATH are possible, including M-A-T-H? **3.** ________

4. In a tennis tournament with 10 players, each player competes in exactly one match with every other player. How many matches are played? **4.** ________

Compute each quotient.

5. $350{,}000 \div 500 =$ ________

6. $2{,}400{,}000 \div 8{,}000 =$ ________

7. $453{,}618 \div 9 =$ ________

8. $12{,}362{,}400 \div 6 =$ ________

9. $1{,}450 \div 5 =$ ________

10. $345 \div 15 =$ ________

Are you ready for Beast Academy 4C?

First, estimate each quotient. Then, compute the quotient and remainder.

11. $198 \div 23$ is closest to: (circle one)

2 10 50 200

$23\overline{)198}$

quotient = _____

remainder = _____

12. $9{,}570 \div 47$ is closest to: (circle one)

2 10 50 200

$47\overline{)9{,}570}$

quotient = _____

remainder = _____

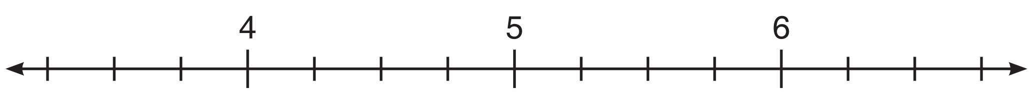
13. Circle the numbers below that are divisible by 4, box the numbers that are divisible by 25, and underline the numbers that are divisible by 100.

187 275 300 360 535 600 625 708 1,000

14. Circle all of the numbers below that 2,375 is divisible by.

2 4 5 12 25 50 1,000

15. Label $\frac{17}{4}$ on the number line below as a mixed number.



Place a <, >, or = to compare each pair of fractions below.

16. $\frac{3}{7} \bigcirc \frac{4}{7}$

17. $\frac{3}{11} \bigcirc \frac{3}{13}$

18. $\frac{5}{17} \bigcirc \frac{10}{37}$

19. Write each of the digits 1 through 8 exactly once in the grid below so that there are two digits in each row and two digits in each column. A number to the left of the grid gives the product of the two numbers in that row. Similarly, a number above the grid gives the product of the two numbers in that column.

	10	8		24
2				
24				
20				

Are you ready for Beast Academy 4C?

Solutions

1. Each number in this list is a multiple of 2. To make the list easier to count, we divide each number in the list by 2.

$$\begin{array}{ccccccc} 6, & 8, & 10, & \ldots, & 50, & 52, & 54 \\ \div 2 & \div 2 & \div 2 & \ldots & \div 2 & \div 2 & \div 2 \\ \hline 3, & 4, & 5, & \ldots, & 25, & 26, & 27 \end{array}$$

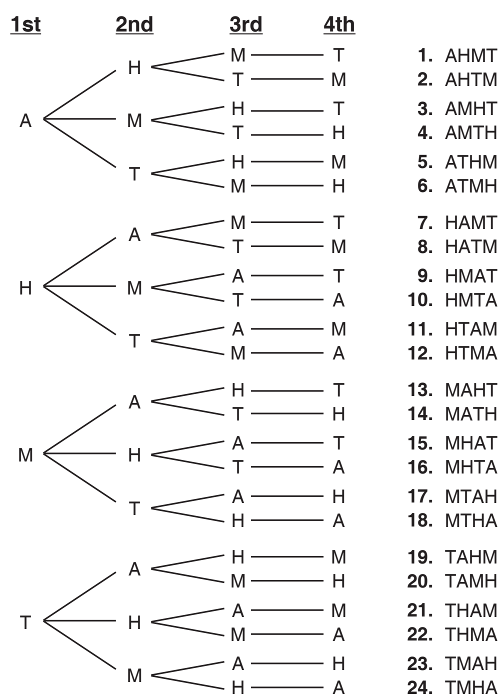
Subtracting 2 from each number in the list gives us a list from 1 to 25.

Now, the numbers are counted for us! So, there are **25** even numbers from 6 to 54.

2. There are five odd digits: 1, 3, 5, 7, and 9. So, to make a three-digit number with three odd digits, we have 5 choices for the hundreds digit, 5 choices for the tens digit, and 5 choices for the ones digit.

All together, there are $5\times5\times5=\mathbf{5^3}=\mathbf{125}$ three-digit numbers with three odd digits.

3. We create a diagram to organize which letter comes first, second, third, and fourth in the arrangement.



All together, there are **24** ways to arrange these letters.

— ***or*** —

We count the arrangements without listing them all. We begin by choosing one letter to be the first. There are 4 choices for the first letter (M, A, T, or H).

Then, no matter which letter we chose as the first, there are always 3 choices for the second letter.

Similarly, no matter which two letters we chose to be first and second, there are always 2 choices remaining for the third letter.

Finally, once we have chosen the first three letters, there is only 1 remaining letter to be the fourth.

All together, there are $4\times3\times2\times1=\mathbf{24}$ ways to arrange the four letters in some order. You may have also written this as **4!** or "**4 factorial**."

4. Each game is played by one pair of players. In choosing a pair of players, we have 10 choices for the "first" player and 9 choices for the "second" player.

Since "Winnie vs. Alex" is the same game as "Alex vs. Winnie," the order in which we pick the players does not matter. So, $10\times9=90$ counts each game twice.

Therefore, there are $90\div2=\mathbf{45}$ games in the tournament.

5. We can divide 3,500 hundreds into groups of 5 hundreds to make $3{,}500\div5=700$ groups of 5 hundreds.

So, $350{,}000\div500=3{,}500\div5=\mathbf{700}$.

6. $2{,}400{,}000\div8{,}000=2{,}400\div8=\mathbf{300}$.

7. Consider dividing 453,618 marbles into 9 buckets.

We begin by dividing 450,000 of the marbles into 9 buckets, which gives us $450{,}000\div9=50{,}000$ marbles in each bucket.

Then, we divide 3,600 more marbles among the 9 buckets, which gives us $3{,}600\div9=400$ more marbles in each bucket.

Finally, we divide the 18 remaining marbles among the 9 buckets, which gives us $18\div9=2$ more marbles in each bucket.

So, each bucket has a total of $50{,}000+400+2=50{,}402$ marbles.

Therefore, $453{,}618\div9=\mathbf{50{,}402}$.

— ***or*** —

$$\begin{aligned} 453{,}618\div9 &= (450{,}000+3{,}600+18)\div9 \\ &= (450{,}000\div9)+(3{,}600\div9)+(18\div9) \\ &= 50{,}000+400+2 \\ &= \mathbf{50{,}402}. \end{aligned}$$

8.
$$\begin{aligned} 12{,}362{,}400\div6 &= (12{,}000{,}000+360{,}000+2{,}400)\div6 \\ &= (12{,}000{,}000\div6)+(360{,}000\div6)+(2{,}400\div6) \\ &= 2{,}000{,}000+60{,}000+400 \\ &= \mathbf{2{,}060{,}400}. \end{aligned}$$

9. In a division problem, we can double both the dividend and the divisor without changing the quotient.

$1{,}450\div5=(1{,}450\times2)\div(5\times2)=2{,}900\div10=\mathbf{290}$.

Remember that fractions are another way to write division, and we can use multiplication to write equivalent fractions:

$$1{,}450\div5=\frac{1{,}450}{5}\underset{\times2}{\overset{\times2}{=}}\frac{2{,}900}{10}=2{,}900\div10=290.$$

10. $345\div15=(345\times2)\div(15\times2)=690\div30=\mathbf{23}$.

Are you ready for Beast Academy 4C?

You may have taken different steps to find the same estimate, quotient, and remainder.

11. 198 is close to 200, and 23 is close to 20. So, we estimate that $198 \div 23$ is about $200 \div 20 = \mathbf{10}$.

$23 \times 5 = 115$. So, 23 can go into 198 at least 5 times. We subtract 115 from 198 and have 83 left over. Since 83 is more than 23, we keep dividing.

$$\begin{array}{r} 5 \\ 23\overline{)\,198} \\ -115 \\ \hline 83 \end{array}$$

$23 \times 3 = 69$, so 23 can go into 83 at least 3 times. We subtract 69 from 83 and have 14 left over. Since 14 is less than 23, we can't subtract any more 23's.

All together, we subtracted $3 + 5 = 8$ 23's.

$$\begin{array}{r} 3 \\ 5 \\ 23\overline{)\,198} \\ -115 \\ \hline 83 \\ -69 \\ \hline 14 \end{array} \left.\begin{array}{l} \\ \\ \end{array}\right\} 8$$

So, the quotient of $198 \div 23$ is **8** and the remainder is **14**.

12. 9,570 is somewhat close to 10,000, and 47 is close to 50. So, we estimate that $9{,}570 \div 47$ is about $10{,}000 \div 50 = \mathbf{200}$.

$47 \times 200 = 9{,}400$. So, 47 can go into 9,570 at least 200 times. We subtract 9,400 from 9,570 and have 170 left over. Since 170 is more than 47, we keep dividing.

$$\begin{array}{r} 200 \\ 47\overline{)\,9{,}570} \\ -9{,}400 \\ \hline 170 \end{array}$$

$47 \times 3 = 141$, so 47 can go into 170 at least 3 times. We subtract 141 from 170 and have 29 left over. Since 29 is less than 47, we can't subtract any more 47's.

$$\begin{array}{r} 3 \\ 200 \\ 47\overline{)\,9{,}570} \\ -9{,}400 \\ \hline 170 \\ -141 \\ \hline 29 \end{array} \left.\begin{array}{l} \\ \\ \end{array}\right\} 203$$

All together, we subtracted $200 + 3 = 203$ forty-sevens.

So, the quotient of $9{,}570 \div 47$ is **203** and the remainder is **29**.

13. A number is divisible by 4 if and only if the number formed by its last two digits is a multiple of 4. The numbers whose last two digits form a multiple of 4 are 300, 360, 600, 708, and 1,000.

A number is divisible by 25 if and only if its last two digits are 00, 25, 50, or 75. The numbers whose last two digits are 00, 25, 50, or 75 are 275, 300, 600, 625, and 1,000.

A number is divisible by 100 if and only if its last two digits are 00. The numbers whose last two digits are 00 are 300, 600, and 1,000.

After circling the numbers divisible by 4, boxing the numbers divisible by 25, and underlining the numbers divisible by 100, we have

187 $\boxed{275}$ $\boxed{300}$ 360 535 $\boxed{600}$ $\boxed{625}$ 708 $\boxed{1{,}000}$

14. The units digit of 2,375 is not 0, 2, 4, 6, or 8, so 2,375 is not divisible by 2.

$4 = 2 \times 2$, so any multiple of 4 is a multiple of 2 and therefore even. Since 2,375 is not even, 2,375 is not divisible by 4.

2,375 ends in 5, so 2,375 is divisible by 5.

$12 = 2 \times 6$, so any multiple of 12 is a multiple of 2 and therefore even. Since 2,375 is not even, 2,375 is not divisible by 12.

2,375 ends in 75, so 2,375 is divisible by 25.

$50 = 2 \times 25$, so any multiple of 50 is a multiple of 2 and therefore even. Since 2,375 is not even, 2,375 is not divisible by 50.

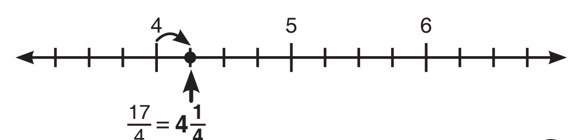
2,375 does not end in 000, so 2,375 is not divisible by 1,000.

After circling the numbers that 2,375 is divisible by, we have

2 4 5 12 25 50 50 1,000

15. We know $\frac{17}{4}$ is between $\frac{16}{4} = 4$ and $\frac{20}{4} = 5$. $\frac{17}{4}$ is one fourth more than $\frac{16}{4} = 4$. So, written as a mixed number, $\frac{17}{4} = 4 + \frac{1}{4} = 4\frac{1}{4}$.

We count one fourth past 4 to reach $4\frac{1}{4} = \frac{17}{4}$.



16. *Three* sevenths is less than *four* sevenths. So, $\frac{3}{7} < \frac{4}{7}$.

17. Elevenths are greater than thirteenths. So, 3 elevenths are greater than 3 thirteenths: $\frac{3}{11} > \frac{3}{13}$.

18. We compare $\frac{5}{17}$ to $\frac{10}{37}$ by converting $\frac{5}{17}$ to an equivalent fraction with numerator 10.

$$\frac{5}{17} \xrightarrow{\times 2} = \frac{10}{34}$$

$\frac{10}{34} > \frac{10}{37}$, so $\frac{5}{17} > \frac{10}{37}$.

— ***or*** —

You may have converted $\frac{5}{17}$ and $\frac{10}{37}$ to equivalent fractions with the same denominator: $17 \times 37 = 629$.

$$\frac{5}{17} \xrightarrow{\times 37} = \frac{185}{629} \quad \text{and} \quad \frac{10}{37} \xrightarrow{\times 17} = \frac{170}{629}$$

$\frac{185}{629} > \frac{170}{629}$, so $\frac{5}{17} > \frac{10}{37}$.

Are you ready for Beast Academy 4C?

19. First, we look for ways to make each product using two different numbers from 1 to 8:

$10 = 2\times5$ $\quad 8 = 1\times8$ or 2×4 $\quad 24 = 3\times8$ or 4×6
$2 = 1\times2$ $\quad 24 = 3\times8$ or 4×6 $\quad 20 = 4\times5$

Since so many of the rows and columns have two possible products, we begin this problem by trying to sort out which products will be used.

We record the possible products on our puzzle as shown below.

		2×5	1×8 or 2×4		3×8 or 4×6
		10	8		24
1×2	2				
3×8 or 4×6	24				
4×5	20				

Since the fourth row's product can only be $20 = 4\times5$, neither 4 nor 5 can be in any other row. So, in the third row, we must use $24 = 3\times8$.

Since the first column's product can only be $10 = 2\times5$, neither the 2 nor 5 can be in any other column. So, in the second column, we must use $8 = 1\times8$. Then, neither the 1 nor 8 can be in any other column. So, we must use $24 = 4\times6$ in the fourth column.

		2×5	1×8 or ~~2×4~~		~~3×8~~ or 4×6
		10	8		24
1×2	2				
3×8 or ~~4×6~~	24				
4×5	20				

Each of the numbers from 1 to 8 must appear once in the puzzle. From these products, we see that the product of the unlabeled third column must be $3\times7 = 21$, and the product of the unlabeled third row must be $6\times7 = 42$.

Now, each number is listed in its row and column, so we complete the puzzle as shown.

		2×5	1×8 or ~~2×4~~	3×7	~~3×8~~ or 4×6
		10	8	21	24
1×2	2	**2**	**1**		
6×7	42			**7**	**6**
3×8 or ~~4×6~~	24		**8**	**3**	
4×5	20	**5**			**4**