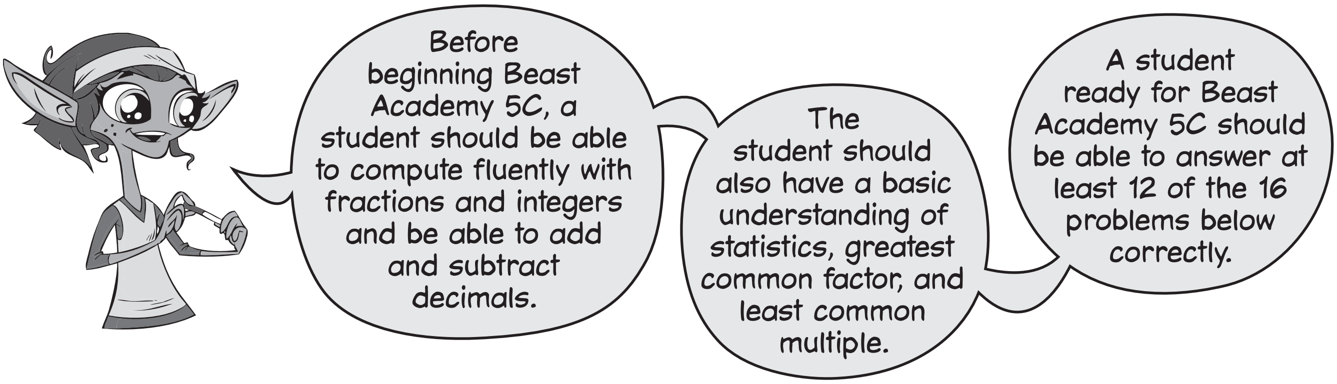
Are you ready for Beast Academy 5C?



Step 1. The student should try to answer every question without a calculator and without help.

Step 2. Check the student's answers using the solutions at the end of this document.

Step 3. The student should be given a second chance on problems that he or she answered incorrectly.

Evaluate each expression below.

1. $4.372 + 11.91 =$ ________

2. $8.36 - 1.058 =$ ________

3. $3\frac{5}{6} + 2\frac{3}{10} =$ _____

4. $4\frac{1}{2} - \frac{3}{5} =$ _____

5. $\frac{5}{18} \cdot \frac{12}{35} =$ _____

6. $3\frac{4}{7} \div \frac{5}{8} =$ _____

7. Order the numbers below from least to greatest. **7.** ______, ______, ______, _______

$4\frac{3}{10}$ 4.037 $\frac{437}{100}$ 4.307

8. **a.** What is the greatest common factor (GCF) of 48 and 90? **a.** GCF: _______

b. What is the least common multiple (LCM) of 48 and 90? **b.** LCM: _______

9. **a.** What is the GCF of 36, 54, and 60? **a.** GCF: _______

b. What is the LCM of 36, 54, and 60? **b.** LCM: _______

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For problems 10-12, use the given numbers to fill in the blanks so that
- ***each statement is true, and***
- ***each fraction is in simplest form.***

10. Numbers: 3, 5, 9, 20

$$\frac{\_\_}{6} \cdot \frac{\_\_}{\_\_} = \frac{\_\_}{8}$$

11. Numbers: 3, 5, 6, 14

$$\frac{\_\_}{\_\_} \cdot \frac{4}{\_\_} = \frac{\_\_}{35}$$

12. Numbers: 5, 12, 20, 30

$$\frac{3}{\_\_} + \frac{\_\_}{\_\_} = \frac{17}{\_\_}$$

Use the prime factorization below to help you answer problems 13 and 14.

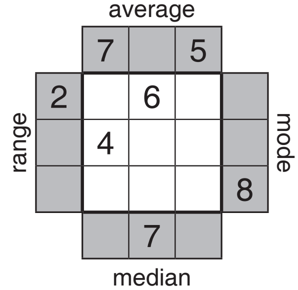
$$159{,}600 = 2^4 \cdot 3 \cdot 5^2 \cdot 7 \cdot 19$$

13. What is the smallest positive integer that we can multiply 159,600 by to get a product that is a perfect square? **13.** ________

14. What is the smallest positive integer that is not a factor of 159,600? **14.** ________

15. A wheelbarrow contains five 6-pound pumpkins and some 19-pound pumpkins. If the average weight of a pumpkin in the wheelbarrow is 14 pounds, how many 19-pound pumpkins are in the wheelbarrow? **15.** ________

16. Fill each empty white square below with a ***positive digit*** so that the clues given in the surrounding shaded squares give the correct average, mode, median, and range for the row or column they label.



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Solutions

1. $4.372 + 11.91 = \mathbf{16.282}$.

2. $8.36 - 1.058 = \mathbf{7.302}$.

3. $3\frac{5}{6} + 2\frac{3}{10} = 3\frac{25}{30} + 2\frac{9}{30} = 5\frac{34}{30} = 6\frac{4}{30} = \mathbf{6\frac{2}{15}} = \mathbf{\frac{92}{15}}$.

4. $4\frac{1}{2} - \frac{3}{5} = 4\frac{5}{10} - \frac{6}{10} = 3\frac{15}{10} - \frac{6}{10} = \mathbf{3\frac{9}{10}} = \mathbf{\frac{39}{10}}$.

5. We first divide 5 and 35 by their greatest common factor, 5.

$$\frac{\cancel{5}^{1}}{18} \cdot \frac{12}{\cancel{35}_{7}}$$

We then divide 12 and 18 by their greatest common factor, 6.

$$\frac{\cancel{5}^{1}}{\cancel{18}_{3}} \cdot \frac{\cancel{12}^{2}}{\cancel{35}_{7}}$$

Now that we have canceled all common factors, we compute the product.

$$\frac{\cancel{5}^{1}}{\cancel{18}_{3}} \cdot \frac{\cancel{12}^{2}}{\cancel{35}_{7}} = \frac{1 \cdot 2}{3 \cdot 7} = \mathbf{\frac{2}{21}}$$

6. $3\frac{4}{7} = \frac{25}{7}$. So, $3\frac{4}{7} \div \frac{5}{8} = \frac{25}{7} \div \frac{5}{8} = \frac{25}{7} \cdot \frac{8}{5} = \frac{\cancel{25}^{5}}{7} \cdot \frac{8}{\cancel{5}_{1}} = \mathbf{\frac{40}{7}} = \mathbf{5\frac{5}{7}}$.

7. We write each fraction as a decimal.

$$4\frac{3}{10} = 4.3 \qquad 4.037 \qquad \frac{437}{100} = 4.37 \qquad 4.307$$

Then, we compare the decimals:

$$4.037 < 4.3 < 4.307 < 4.37.$$

Writing the original numbers in order from least to greatest, we have

$$\mathbf{4.037,\ 4\frac{3}{10},\ 4.307,\ \frac{437}{100}}.$$

— or —

We write each number as a mixed number with a denominator of 1,000.

$$4\frac{3}{10} = 4\frac{300}{1{,}000} \qquad 4.037 = 4\frac{37}{1{,}000}$$

$$\frac{437}{100} = 4\frac{37}{100} = 4\frac{370}{1{,}000} \qquad 4.307 = 4\frac{307}{1{,}000}$$

Then, we compare the mixed numbers:

$$4\frac{37}{1{,}000} < 4\frac{300}{1{,}000} < 4\frac{307}{1{,}000} < 4\frac{370}{1{,}000}.$$

Writing the original numbers in order from least to greatest, we have

$$\mathbf{4.037,\ 4\frac{3}{10},\ 4.307,\ \frac{437}{100}}.$$

8. **a.** We use the prime factorizations of 48 and 90.

$$48 = 2^4 \cdot 3 \quad \text{and} \quad 90 = 2 \cdot 3^2 \cdot 5$$

The GCF of 48 and 90 is the product of all the prime factors they share. So, the GCF is $2 \cdot 3 = \mathbf{6}$.

b. To compute the LCM, we take the largest power of each prime in either number's prime factorization.

$$48 = 2^4 \cdot 3 \quad \text{and} \quad 90 = 2 \cdot 3^2 \cdot 5$$

So, the LCM of 48 and 90 is $2^4 \cdot 3^2 \cdot 5 = \mathbf{720}$.

9. **a.** We use the prime factorizations of 36, 54, and 60.

$$36 = 2^2 \cdot 3^2 \qquad 54 = 2 \cdot 3^3 \qquad 60 = 2^2 \cdot 3 \cdot 5$$

The GCF of 36, 54, and 60 is the product of all the prime factors they share. So, the GCF is $2 \cdot 3 = \mathbf{6}$.

b. To compute the LCM, we take the largest power of each prime in the numbers' prime factorizations.

$$36 = 2^2 \cdot 3^2 \qquad 54 = 2 \cdot 3^3 \qquad 60 = 2^2 \cdot 3 \cdot 5$$

So, the LCM of 36, 54, and 60 is $2^2 \cdot 3^3 \cdot 5 = \mathbf{540}$.

10. All fractions must be in simplest form. Among our choices, only 5 can be the numerator of the fraction with denominator 6.

$$\mathbf{\frac{5}{6}} \cdot \frac{\quad}{\quad} = \frac{\quad}{8}$$

The remaining numbers are 3, 9, and 20.

Since the denominator of the product is not a multiple of 3, the factor of 3 in the denominator of $\frac{5}{6}$ must cancel. So, the numerator of the middle fraction is a multiple of 3.

3 or 9

$$\frac{5}{6} \cdot \frac{\quad}{\quad} = \frac{\quad}{8}$$

Among our choices, only 3 and 9 are multiples of 3.

Since all fractions are in simplest form, 20 cannot be the numerator of $\frac{\ }{8}$. Therefore, 20 is the denominator of the middle fraction.

3 or 9

$$\frac{5}{6} \cdot \frac{\quad}{\mathbf{20}} = \frac{\quad}{8}$$

Finally, we place 3 and 9 in the empty numerators in the only way that makes a true statement.

$$\frac{5}{6} \cdot \frac{\mathbf{3}}{\mathbf{20}} = \frac{\mathbf{9}}{8}$$

Check: $\frac{\cancel{5}^{1}}{\cancel{6}_{2}} \cdot \frac{\cancel{9}^{3}}{\cancel{20}_{4}} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$. ✓

11. The denominator of the product is $35 = 5 \cdot 7$. So, the denominators of the two fractions that we multiply must include at least one multiple of 5 and one multiple of 7.

Among our choices, only 5 is a multiple of 5, and only 14 is a multiple of 7. These two numbers are therefore the denominators of the fractions that we multiply.

Since all fractions are in simplest form, 14 cannot be the denominator of $\frac{4}{\ }$. We place 5 and 14 in the empty denominators as shown.

$$\frac{\quad}{\mathbf{14}} \cdot \frac{4}{\mathbf{5}} = \frac{\quad}{35}$$

The remaining numbers are 3 and 6.

$$\frac{\mathbf{3}}{\mathbf{14}} \cdot \frac{4}{\mathbf{5}} = \frac{\mathbf{6}}{35}$$

All fractions are in simplest form, so we place the 3 and 6 in the empty numerators as shown.

Check: $\frac{3}{\cancel{14}_{7}} \cdot \frac{\cancel{4}^{2}}{5} = \frac{3 \cdot 2}{7 \cdot 5} = \frac{6}{35}$. ✓

12. Both the numerator and denominator of the middle fraction are empty.

5 and 12

$$\frac{3}{\quad} + \frac{\quad}{\quad} = \frac{17}{\quad}$$

All fractions are in simplest form. Among our four number choices, 5 and 12 are the only pair with a GCF of 1. So, only 5 and 12 can be the numerator and denominator of the middle fraction.

The remaining numbers are 20 and 30.

Since each fraction is in simplest form, we can only place 20 and 30 in the empty denominators as shown.

5 and 12

$$\frac{3}{\mathbf{20}}+\frac{}{}=\frac{17}{\mathbf{30}}$$

Then, since the sum $\frac{17}{30}$ is less than 1 and we are adding only positive numbers, we know the middle fraction is also less than 1. So, we place the 5 and 12 as shown.

$$\frac{3}{\mathbf{20}}+\frac{\mathbf{5}}{\mathbf{12}}=\frac{17}{\mathbf{30}}$$

Check: $\frac{3}{20}+\frac{5}{12}=\frac{9}{60}+\frac{25}{60}=\frac{34}{60}=\frac{17}{30}$. ✓

13. If a number is a perfect square, then it can be written as the product of two identical groups of prime factors. So, in the prime factorization of a perfect square, every prime has an even exponent.

In the prime factorization of $159{,}600=2^4\cdot3\cdot5^2\cdot7\cdot19$, the primes 3, 7, and 19 each have odd exponents (3^1, 7^1 and 19^1). So, multiplying 159,600 by $3\cdot7\cdot19$ gives a perfect square:

$$\begin{aligned}159{,}600\cdot3\cdot7\cdot19 &= (2^4\cdot3\cdot5^2\cdot7\cdot19)\cdot3\cdot7\cdot19\\ &= 2^4\cdot3^2\cdot5^2\cdot7^2\cdot19^2\\ &= (2^2\cdot3\cdot5\cdot7\cdot19)\cdot(2^2\cdot3\cdot5\cdot7\cdot19)\\ &= 7{,}980\cdot7{,}980\\ &= 7{,}980^2.\end{aligned}$$

Multiplying 159,600 by any integer less than $3\cdot7\cdot19$ would result in a product that is not a perfect square. So, $3\cdot7\cdot19=\mathbf{399}$ is the smallest positive integer we can multiply 159,600 by to make a perfect square.

14. If an integer x is not a factor of 159,600, then either

- x has at least one prime factor that is not a factor of 159,600, ***or***
- the power of a prime factor x is larger than its power in 159,600.

The smallest prime factor that is not in $2^4\cdot3\cdot5^2\cdot7\cdot19$ is 11.
The smallest power of 2 larger than 2^4 is $2^5=32$.
The smallest power of 3 larger than 3 is $3^2=9$.
The smallest power of 5 larger than 5^2 is $5^3=125$.
The smallest power of 7 larger than 7 is $7^2=49$.
The smallest power of 19 larger than 19 is $19^2=361$.

Among the possibilities above, the smallest is $3^2=9$.
So, **9** is the smallest positive integer that is not a factor of 159,600.

— ***or*** —

Each integer from 1 to 8 is a factor of $2^4\cdot3\cdot5^2\cdot7\cdot19$:

$\underline{1}$: $2^4\cdot3\cdot5^2\cdot7\cdot19=1\cdot(2^4\cdot3\cdot5^2\cdot7\cdot19)$
$\underline{2}$: $2^4\cdot3\cdot5^2\cdot7\cdot19=2\cdot(2^3\cdot3\cdot5^2\cdot7\cdot19)$
$\underline{3}$: $2^4\cdot3\cdot5^2\cdot7\cdot19=3\cdot(2^4\cdot5^2\cdot7\cdot19)$
$\underline{4}$: $2^4\cdot3\cdot5^2\cdot7\cdot19=4\cdot(2^2\cdot3\cdot5^2\cdot7\cdot19)$
$\underline{5}$: $2^4\cdot3\cdot5^2\cdot7\cdot19=5\cdot(2^4\cdot3\cdot5\cdot7\cdot19)$
$\underline{6}$: $2^4\cdot3\cdot5^2\cdot7\cdot19=6\cdot(2^3\cdot5^2\cdot7\cdot19)$
$\underline{7}$: $2^4\cdot3\cdot5^2\cdot7\cdot19=7\cdot(2^4\cdot3\cdot5^2\cdot19)$
$\underline{8}$: $2^4\cdot3\cdot5^2\cdot7\cdot19=8\cdot(2\cdot3\cdot5^2\cdot7\cdot19)$

However, $9=3^2$ is *not* a factor $2^4\cdot3\cdot5^2\cdot7\cdot19$. So, **9** is the smallest positive integer that is not a factor of 159,600.

15. Each 6-pound pumpkin is 8 pounds below the average, and each 19-pound pumpkin is 5 pounds above the average. The five 6-pound pumpkins are a total of $5\cdot8=40$ pounds below the average. To balance this, we need enough 19-pound pumpkins to equal 40 pounds above the average.

$$\overbrace{\begin{matrix}-8&-8&-8&-8&-8\\6&6&6&6&6\end{matrix}}^{-40}\qquad\qquad\overbrace{\begin{matrix}+5&\cdots&+5\\19&\cdots&19\end{matrix}}^{+40}$$

So, there are $\frac{40}{5}=\mathbf{8}$ nineteen-pound pumpkins in the wheelbarrow.

— ***or*** —

We write and solve an equation. Let n be the number of 19-pound pumpkins in the wheelbarrow.

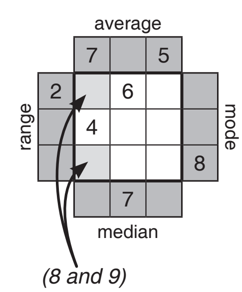
- The five 6-pound pumpkins weigh a total of $5\cdot6=30$ pounds, and the n 19-pound pumpkins weigh a total of $19n$ pounds. So, the total weight of all the pumpkins is $30+19n$.
- There are $5+n$ pumpkins in the wheelbarrow with an average weight of 14 pounds, so their total weight is $14(5+n)=70+14n$.

We have two expressions for the total weight of the pumpkins, so we write an equation:

$$30+19n=70+14n.$$

Solving for n, we get $n=8$. Therefore, there are **8** 19-pound pumpkins in the wheelbarrow.

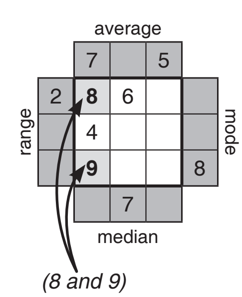
16. Adding all of the numbers in a list and then dividing by the number of numbers gives us their ***average***.



The average of the numbers in the left column is 7, so the sum of these three numbers is $3\cdot7=21$.

There is already a 4 in this column, so the sum of the two remaining numbers is $21-4=17$. The only way to make a sum of 17 from two positive digits is $8+9$.

The ***range*** is the difference between the largest and smallest numbers in a data set.



In the top row, the range is 2. The top row already contains a 6, so it cannot contain any digit larger than $6+2=8$.

So, we place the 8 and 9 in the left column as shown.

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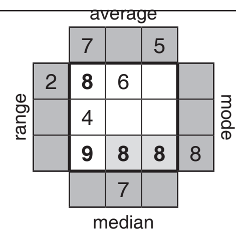
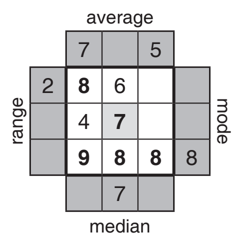
The ***mode*** is the number that appears most in a list.

The mode of the three integers in the bottom row is 8, so this number must appear at least twice in the row. Since we already have a 9 in the bottom row, the two other squares in this row both contain 8.



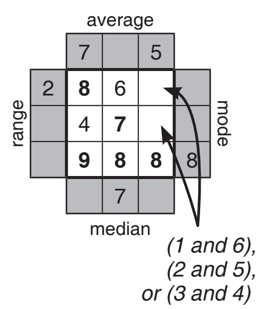
The ***median*** is the number in the middle when we arrange a list in order from least to greatest.

The median of the middle column is 7. Since this column already contains a 6 and 8, the remaining number in this column is 7.



The average of the numbers in the right column is 5, so the sum of these three numbers is $3 \cdot 5 = 15$.

There is already an 8 in this column, so the sum of the two remaining numbers is $15 - 8 = 7$. We can write 7 as the sum of two digits in 3 ways: $1+6$, $2+5$, and $3+4$.



The range in the top row is 2, and the two digits already placed have a range of 2. Therefore, the third number in this top row can only be 6, 7, or 8.

Only one of our three pairs in the right column contains a 6, 7, or 8: $1+6$.

So, we place the 1 and 6 in the right column as shown to complete the puzzle.