PYTHAGOREAN PATHS

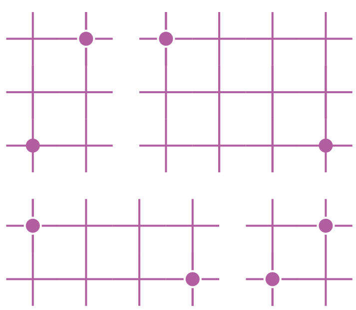
STRATEGIES

1. Figure out how each length can be drawn on the grid.

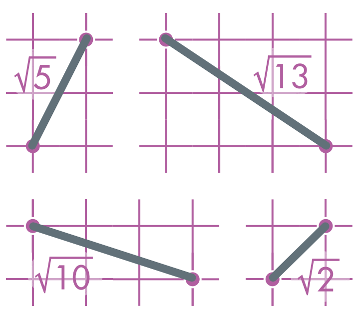
It helps to figure out how each segment length can be drawn on the grid.

Match each segment length to a pair of dots below.

$\sqrt{2}$ $\sqrt{5}$ $\sqrt{10}$ $\sqrt{13}$



We can use the Pythagorean theorem to find the distance between each pair of dots. We label them as shown below.



Before beginning each puzzle, it can be useful to describe each length by the rectangle whose corners it connects:

1×1	1×2	1×3	2×3
$\sqrt{2}$	$\sqrt{5}$	$\sqrt{10}$	$\sqrt{13}$

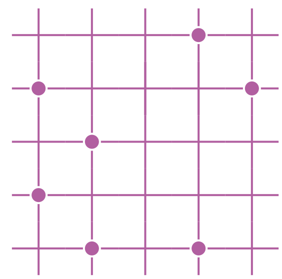
2. Find all of the segments of a given length.

Finding all of the segments of a given length can help us see how they can be part of the path.

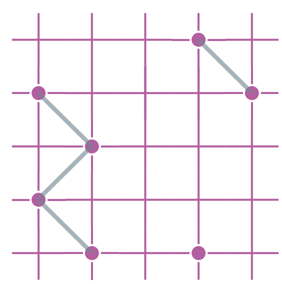
If the number of segments that can be drawn on the grid matches the number of segments on the path, then they all must be part of the path.

How many segments of length $\sqrt{2}$ connect pairs of dots in the grid below?

$\sqrt{2}$ $\sqrt{8}$ $\sqrt{2}$ 2 $\sqrt{2}$ 2



A segment of length $\sqrt{2}$ connects opposite corners of a 1×1 square. We connect the four pairs of dots that are opposite corners of a 1×1 square and look for ways to use them as segments on the path.



The path contains 3 of these segments. Which segment above cannot be part of the path?

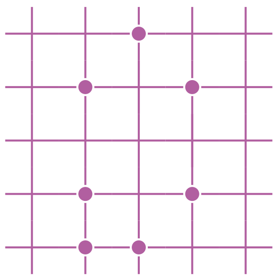
3. Find long or uncommon segments on the grid.

It's usually not a good idea to try to find all of the segments of every length.

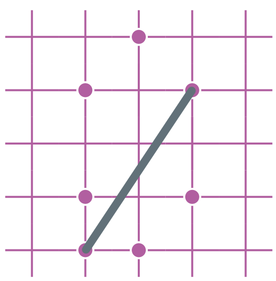
Instead, look for segments that are uncommon or easy to find. These are often the longest segments.

Where can we draw the segment of length $\sqrt{13}$ that is part of the path below?

$\sqrt{2}$ $\sqrt{2}$ $\sqrt{5}$ $\sqrt{13}$ $\sqrt{2}$ $\sqrt{2}$



A segment of length $\sqrt{13}$ connects opposite corners of a 2×3 rectangle. There is only one pair of dots that are at opposite corners of a 2×3 rectangle, so we draw them as shown:



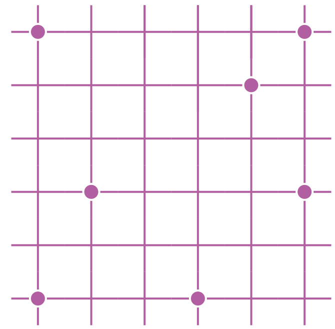
How can we connect segments of length $\sqrt{2}$ and $\sqrt{5}$ to opposite ends of this segment?

4. Start in the middle or the end.

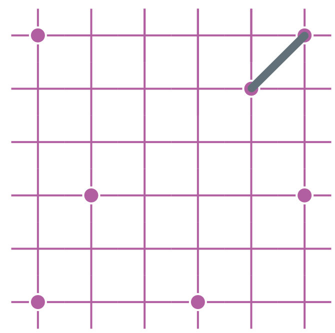
We don't need to start at the beginning of a path. It's often easier to find a segment in the middle or at the end of the path and work from there.

Where is the final segment on the path below?

$\sqrt{5}$ 3 $\sqrt{34}$ $\sqrt{34}$ $\sqrt{5}$ $\sqrt{2}$



There is only one way to draw a segment of length $\sqrt{2}$, as shown below.

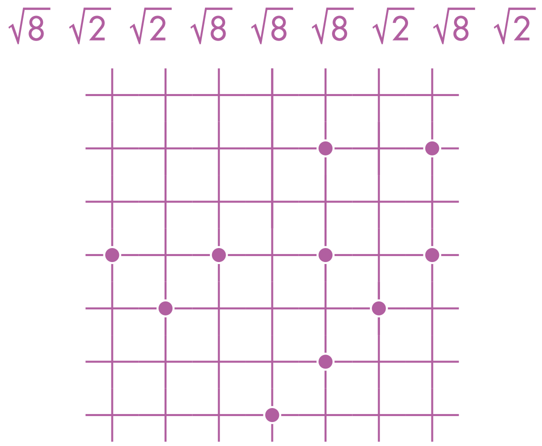


How can we work backwards to continue the path from there?

5. Look for pairs or groups of segment lengths that are easy to find.

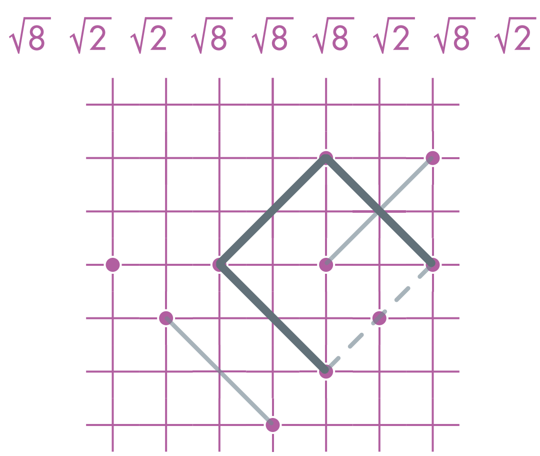
Some paths don't have one segment that stands out as easy to find. Look for pairs or groups of segments that you can find on the grid.

Where are the three segments of length $\sqrt{8}$ that connect in a row to form the middle of the path below?



A segment of length $\sqrt{8}$ connects opposite corners of a 2×2 square. We connect the five pairs of dots that are at opposite corners of a 2×2 square (we can ignore the dashed segment below because it passes through another dot).

Three of these segments are connected, so they must form the middle of the path.

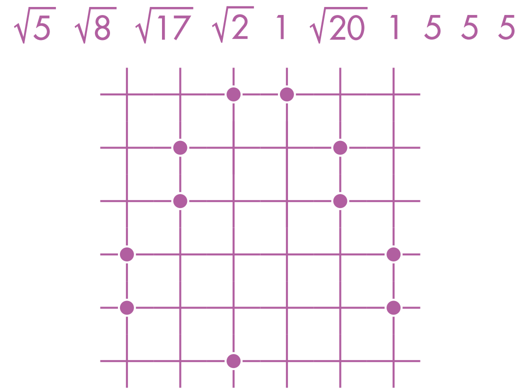


How can we continue the path from both ends of this segment?

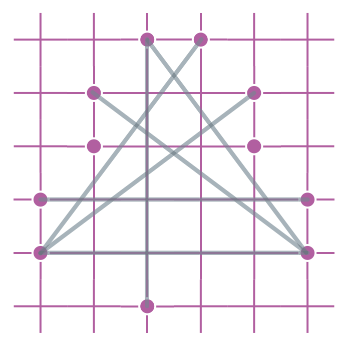
6. Watch out for 5's.

A segment of length 5 can be drawn horizontally or vertically along a grid line, but it can also connect the opposite corners of a 3×4 rectangle.

How many segments of length 5 connect pairs of dots in the grid below?



There are two horizontal segments, one vertical segment, and four segments that connects opposite corners of a 3×4 rectangle for a total of seven segments.

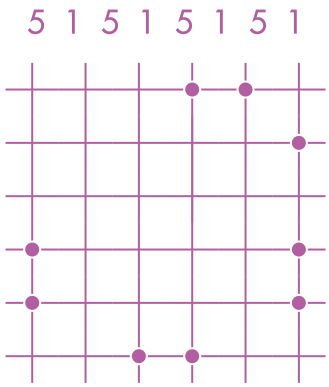


Which three of these connect to form the end of the path?

7. Find dots with limited connections.

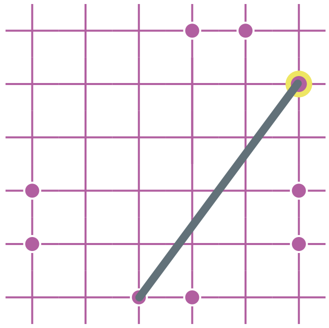
In puzzles with only a few different segment lengths, it can be helpful to look for dots that can't make connections using one or more of these lengths.

Which dot in the puzzle below cannot be connected to another dot by a segment of length 1?



There are four pairs of dots that are one unit apart, but the highlighted dot below cannot be connected to another dot by a segment of length 1.

It can be connected to a dot with a segment of length 5. So, this dot must be the start of the path.



Where does the path go next?